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PROCEEDINGS
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SOME RECENT OBSERVATIONS OF SUNSPOT SPECTRA

By Harold D. Babcock

Mount Wilson Observatory, Carnegie Institution of Washington

Read before the Academy October 25, 1938

Powerful instruments have brought within reach of observation in the visible region a wealth of detail in the sunspot spectrum for comparison with the normal solar spectrum and with laboratory results. The most extensive discussion of this subject is found in the papers of Miss Moore,[1] whose treatment, in the light of the theory of ionization, develops a comprehensive explanation for most of the observed effects.

Corresponding observational data in the infra-red and the ultra-violet are much needed, however, since critical tests for the behavior of certain elements are found only in these regions. Indeed, the only direct evidence for the occurrence of some substances in the sun must be sought in the infra-red spot spectrum, while ultimate lines of the heavy metals lie in the ultra-violet.

The main program of sunspot spectroscopy at this Observatory has been carried out with the tower telescopes and plane-grating spectrographs on Mount Wilson. This equipment is admirably adapted for the visible region of the spectrum and can be used, though with increasing difficulty, in the more accessible part of the infra-red. But its optical limitations become practically prohibitive for the study of spots in the ultra-violet and the more difficult part of the photographic infra-red.

To supplement this combination of instruments a 21-foot concave grating spectrograph has recently been constructed at the Hale Solar Laboratory in Pasadena. The mounting is a slight modification of the Eagle type, with no optical parts except slit and grating. An achromatic solar image, either 40 cm. or 16 cm. in diameter, is provided by a reflecting telescope of 46 cm. aperture, an arrangement well adapted to the study of various solar problems. The spectrograms that have been made to test the capabilities of the equipment include a few plates showing the spectra of the prominent sunspots available in recent weeks.

The large solar image was used, with no nicol prism or other analyzing apparatus at the slit. Under these conditions an Eastman IZ plate hyper-

sensitized with ammonia requires a two-hour exposure for the spot spectrum in the region λλ 10,000–11,200, where the dispersion is about 2.4 A/mm., while for shorter wave-lengths the time of exposure is greatly reduced. Although this dispersion is only about one-third of that given by the 75-foot spectrograph in the first order, it is sufficient for answering many questions. The omission of an analyzer is dictated at present by practical considerations, but is by no means a serious obstacle to progress even in the infra-red, partly because the dispersion is insufficient to show the resolution of the Zeeman effect in any of the spots thus far examined.

In this interesting part of the spot spectrum that is now observed for the first time there is no serious atmospheric absorption over a range of about 1000 A to impair our scrutiny of celestial objects. It is here too that Z plates have a fairly uniform sensitivity and thus facilitate the reconnaissance. The results thus far derived from a preliminary survey of the plates are in excellent accordance with the conclusions of Miss Moore. They may be summarized in a few general descriptive statements.

Lines of the Paschen series of hydrogen are greatly weakened in the spot, as are the principal multiplets of C, O, S and P. Not much change is found for Mg. The strong Si lines are widened and moderately weakened. Definite strengthening is shown by Sr^+, an analogy to the behavior of Ba^+ in the visible region. No evidence appears on these plates for the existence of Cs in the spot spectrum, a confirmation of Miss Moore's conclusion that this element is not observable in the sun. Ca, Fe, Cr and Ni exhibit either weakening, strengthening or no change, just as in the visible part of the spectrum, in accordance with the excitation potentials.

One of the most striking features of the infra-red spot spectrum is the enormous enhancement of many Ti lines. These will probably be found, on further comparison, to strengthen as much as any metallic lines in other spectral regions, e.g., the red lines of V.

A number of new lines, not thus far seen in the spectrum of the solar disc, are now found in the spot. These are apparently of atomic origin; no new bands have been observed.

It is well known that the chief obstacle in the study of the sunspot spectrum in the ultra-violet is the atmospheric and instrumental scattering of light from the disc. Although the Hale Solar Laboratory is located at an altitude of only about 700 feet above sea-level the new apparatus described above has proved distinctly useful for the observation of the ultra-violet region of the solar spectrum. Under observing conditions plainly below the best, a few spectrograms of sunspots have been made on which numerous effects can be seen as far as λ 3100. These make it clear that the spectrum of ultra-violet light scattered from the solar disc does not completely mask that of the umbra itself, though without doubt the spot spectrum is con-

taminated. Exposures five or six times as long as for the disc are required to attain the same photographic density in the spot.

The ultra-violet plates thus far obtained show many interesting effects. Band lines identified with the molecules OH and NH are stronger in the spot than in the disc, but the strong CN bands show no change. Large numbers of metallic lines are intensified in the spot and a few are perceptibly weakened, while many show no definite change from their appearance in the disc. The weakened lines thus far noted in the ultra-violet are those of ionized elements, e.g., Ni^+. Among the most noticeably strengthened lines are low level multiplets of Ti and V, as would be expected. The behavior of a few heavy elements is particularly interesting, especially Ag, Cd, Rh, Ru and Pd, whose ultimate or penultimate lines are undoubtedly strengthened in the spot. No evidence is found in either disc or spot for the presence of Tl, whose ultimate line is at λ 3775.

Much detail awaits examination on the plates already made, and, with some further instrumental improvements under way, new sunspots of large diameter will be eagerly awaited.

[1] *Atomic Lines in the Sunspot Spectrum.* Princeton (1933). Mt. Wilson Contr. No. 446, *Astrophys. Journ.*, **75**, 222 (1932).

SECOND NOTE ON A METAGALACTIC DENSITY GRADIENT

BY HARLOW SHAPLEY

HARVARD COLLEGE OBSERVATORY

Communicated November 15, 1938

1. An earlier note[1] on large non-uniformities in the metagalaxy presented evidence that in a zone thirty degrees wide, extending for 125 degrees across the south galactic polar cap, the density of population of eighteenth magnitude galaxies varies conspicuously. In addition to local irregularities of two or three hundred per cent from one area of nine square degrees to the next, there is a general tendency toward increasing population from a region in Aquarius at galactic latitude $-60°$, longitude $45°$, to a region in Pictor, at galactic latitude $-35°$, longitude $225°$, some two hundred million light years distant. It appears beyond doubt that this gradient is an important structural feature of the metagalaxy, and cannot be attributed to variations in space absorption.

The existence of inequalities in nebular density within limited areas of a few tens of degrees is easily shown and quantitatively measured on the large-field Harvard plates. There is need for caution, however, in attempting to obtain quantitative measures of the long-range gradient, referred to

above, which extends over a considerable interval in both right ascension and declination. One uncertainty in making a census of faint galaxies arises from uncertainties in the stellar photographic standards upon which we must base our reduction of nebular counts to a common distance and magnitude limit.

For the tabular and graphical results of the earlier note, the nebular counts were reduced in the best way that is at present available. They were brought to a common stellar magnitude limit by means of the well-known tables of Seares and van Rhijn, which give the relation between star numbers, apparent photographic magnitudes and galactic coördinates.

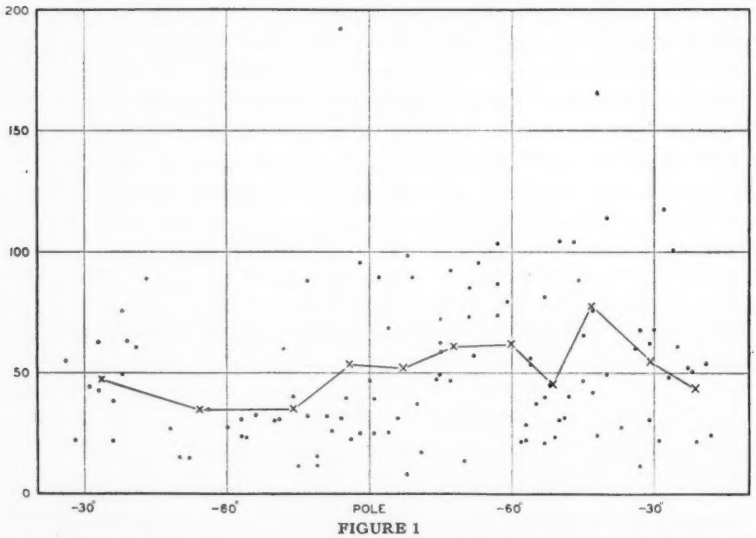

FIGURE 1

Distribution of galaxies in zone across the South Galactic Polar Cap. Ordinates are numbers per square degree; abscissae, distance in degrees along the zone.

The procedure has been pretty well standardized. On the three-hour Bruce plates the magnitude sequences, used in the determination of the magnitude limits for galaxies, have been established by the star-count method, which has proved to be highly satisfactory in northern declinations (north of $\delta = -20°$) where the basic Selected Area sequences have been extensively studied by Seares and his collaborators at Mount Wilson. In southern declinations, however, the basic Selected Area sequences are as yet insecurely established, and the tables are therefore less dependable. There is consequently some risk in the reduction of the nebular counts to a common magnitude limit by this method.

That such procedure is at times precarious is indicated by the fact that groups of Bruce plates taken with identical exposure time, zenith distance and general observing conditions occasionally show a variation in plate limit apparently dependent on galactic coördinates but actually resulting from peculiarities of the van Rhijn tables.

2. To free the evidence for a metagalactic density gradient from the suspected errors of the existing star-count tables, we need only to work with the *unreduced* numbers of galaxies on the individual plates. The following tabulation gives the material in summarized form. Each entry represents ten plates, covering ninety square degrees. In this study of the long-range inequality, the local irregularities are, of course, largely smoothed out by the grouping. Figure 1 again shows that, independently of the stellar magnitude system, there is a strong metagalactic gradient.

TABLE 1

THE DISTRIBUTION OF FIFTY THOUSAND GALAXIES

GALACTIC LONGITUDE °	GALACTIC LATITUDE °	NUMBER PER SQUARE DEGREE	NUMBER PER SQUARE DEGREE, REDUCED
45	33.8	47.57	37.28
45	54.0	34.65	30.57
45	74.0	35.32	32.68
45	85.9	53.78	32.87
225	82.9	52.19	33.03
225	72.2	61.13	43.41
225	60.1	62.09	65.41
225	51.4	45.59	60.03
225	43.3	77.48	109.44
225	30.9	54.82	75.29
225	21.3*	43.85	66.74

* Six plates are involved in this mean, covering fifty-four square degrees.

The difference in slope between the curve in figure 1 and that given in the earlier note is both conspicuous and important. It affords direct evidence that at the eighteenth magnitude the system of stellar magnitudes in the southern hemisphere—at least along the 45°, 225° longitude circle—is not dependable. Before satisfactory quantitative work on the magnitudes of faint galaxies south of $\delta = -20°$ can be undertaken, there must be a general investigation of the magnitudes of southern Selected Areas, and a subsequent correction or replacement of the star-count tables.

In figure 2 the same material as that of figure 1 is represented, but each plate has been reduced to a common magnitude limit by means of the van Rhijn tables, but without employing the corrections for galactic longitude. That is, for each galactic latitude and apparent magnitude the mean value of the star numbers has been adopted, rather than the published different values for different galactic longitudes. We have thus made use of the star

counts on the photographic plates to indicate their effectiveness in recording galaxies, and have not assumed average equality for all plates. This procedure is intermediate between that used in the earlier communication (unqualified use of the van Rhijn tables) and the procedure above, in which no reduction whatever is made on the basis of star-counts. This intermediate procedure probably gives us the best picture available at the present time. Certainly it gives to the results from the individual plates (plotted as small points in figure 2) more significance than the corresponding points in figure 1, since the unreduced values must include the true

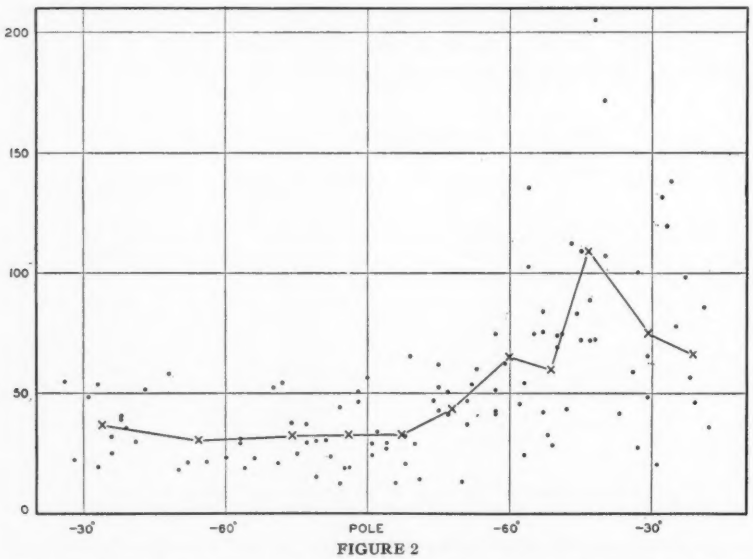

FIGURE 2

Same as figure 1, but the numbers have been reduced to a common magnitude limit.

inequalities in the penetrating power of the individual plates as well as the non-uniformity in the distribution of galaxies.

3. We might inquire as to other factors that could contribute illusory data on the metagalactic gradient. Since the results from the unreduced material summarized above depend on the assumption of average uniformity in plate effectiveness (for means of ten), we should consider the possible changes in emulsion speed, differences in plate quality, changes in transparency with the season and differences in zenith distance at mid-exposure. It is found on close examination that none of these factors contributes appreciably to the production of a spurious gradient. If anything, they have diminished the systematic inequality, as can be inferred by com-

paring figures 1 and 2 of this note. Space is not taken here to report on the details of examining these points, since the program of rectifying the star-count tables is now under way, and we should soon be able to reduce each southern plate directly and satisfactorily to a common magnitude limit and therefore make unnecessary the application of rather uncertain corrections for plate quality, zenith distance and seasonal effects.

In an earlier communication[2] the large inequalities in nebular densities over the south galactic polar cap were discussed. The question arises as to whether the deficiencies in the star-count tables may have contributed erroneous deductions in that study. It is easily seen that, in the mean, the inequalities are essentially the same whether or not the star-number tables are used. Thus we have for the eastern and western halves of the south galactic polar cap the number of galaxies per square degree:

	EAST	WEST
Reduced to $18^m.2$	60.82	43.14
Unreduced	57.36	44.06

These results refer to the central nine square degrees of the eighty plates involved. For the surrounding sixteen square degrees on these plates, we have:

	EAST	WEST
Reduced to $17^m.87$	40.07	27.67
Unreduced	37.52	25.10

In these comparisons, errors in the star-number tables depending on declination do not affect the results, since the intervals of declination involved are the same for both the western and the eastern sections of the polar cap.

[1] These PROCEEDINGS, **24**, 282 (1938).
[2] *Ibid.*, **24**, 148 (1938).

REFLECTIVITY AND COLOR OF METEORITES

By Fletcher G. Watson

Harvard College Observatory

Communicated November 15, 1938

The principal data useful in establishing the physical characteristics of solid extra-terrestrial material, e.g., asteroids, are obtained from studies of spectral-reflectivity (color and albedo), phase effects and degree of polarization of the reflected light. For comparison with the extra-terrestrial bodies the earth-bound investigator has only two types of material: the various telluric stuffs and meteorites. Although the former are the more plentiful, we might expect the latter, of extra-terrestrial origin, to be more comparable to the interplanetary bodies. Reports of only two experiments upon the spectral-reflectivity of meteorites have been found. The first[1] of these is the bare statement that a highly polished and specularly reflecting surface of "Atacama"* gave sixty-four per cent in the green, fifty-nine per cent in the yellow and fifty-eight per cent in the red; thus suggesting that the light is made bluer upon reflection. The second study[2] revealed through photographic spectral-photometry that between λ 4000 and λ 6500 a fresh cleavage surface of Saratov (gray spherical chondrite, fell September 11, 1918, between 4 and 6 P. M.) reflected approximately twenty-five per cent of the incident light with a gray or uniform reflecting power. To make possible extensive comparisons between the extra-terrestrial bodies and the materials which can be examined in the laboratory, more data of this nature, covering a wide variety of meteoritic materials, are greatly to be desired.

The self-recording spectral-photometer in the Color Measurement Laboratory of the Massachusetts Institute of Technology is well adapted to the study of meteoritic reflectivity. Two restrictions are placed by the instrument upon the samples studied: (1) They should be nearly flat, and (2) they should have a surface at least one inch square. To these instrumental restrictions a third is necessarily added: The surface should not show weathering or discoloration from rust. Professor Charles Palache kindly permitted the use of the collection of meteorites at the Harvard University Museum. Many stone specimens in this collection were found to approximate the requirements, and seven were selected as representative of the various types and shades of meteorites. In six cases the surface examined was a broken surface approximately flat; the other sample, Mighei, had a roughly sawn surface. Such surfaces are necessary as we

* The nomenclature of Prior's Catalogue 1923, 1927, has been used throughout. "Atacama" is not a recognized identification, but is a synonym for two metallic meteorites from the Desert of Atacama: Imilac, pallasite; and Copiapo, brecciated octahedrite with silicate inclusions. It seems probable that Copiapo was the mass investigated.

expect the extra-terrestrial bodies to have rough surfaces comparable to those obtained upon fracture. Although the majority of iron meteorites had polished surfaces, a sample of Canyon Diablo was found with a rela-

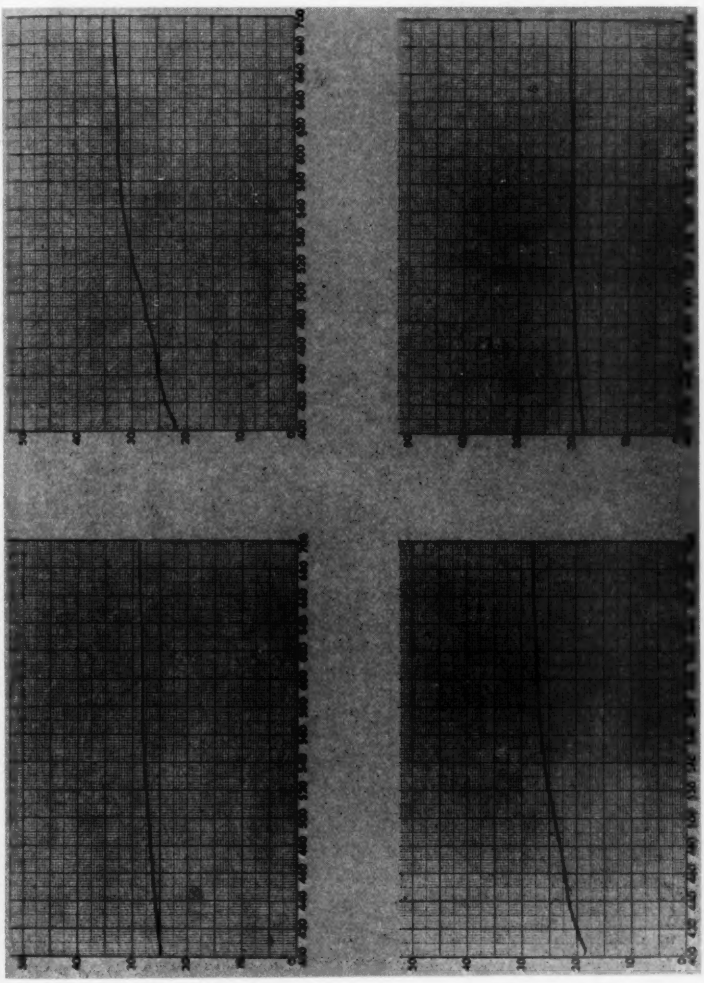

FIGURE 1

Spectral-reflectance curves for meteorites.

Allegan Drake Creek
Waconda Olmedilla

Abscissae are wave-lengths in millemicrons; ordinates, percentage reflectance.

tively rough surface formed by deep etching after superficial polishing. It was examined to determine whether or not metallic and stony materials could be distinguished on the basis of color.

The spectral-reflectivities of these specimens are shown in figures 1 and 2. The descriptions of the surfaces examined are given in table 1. Comparison was made with a magnesium-oxide surface whose reflectance was

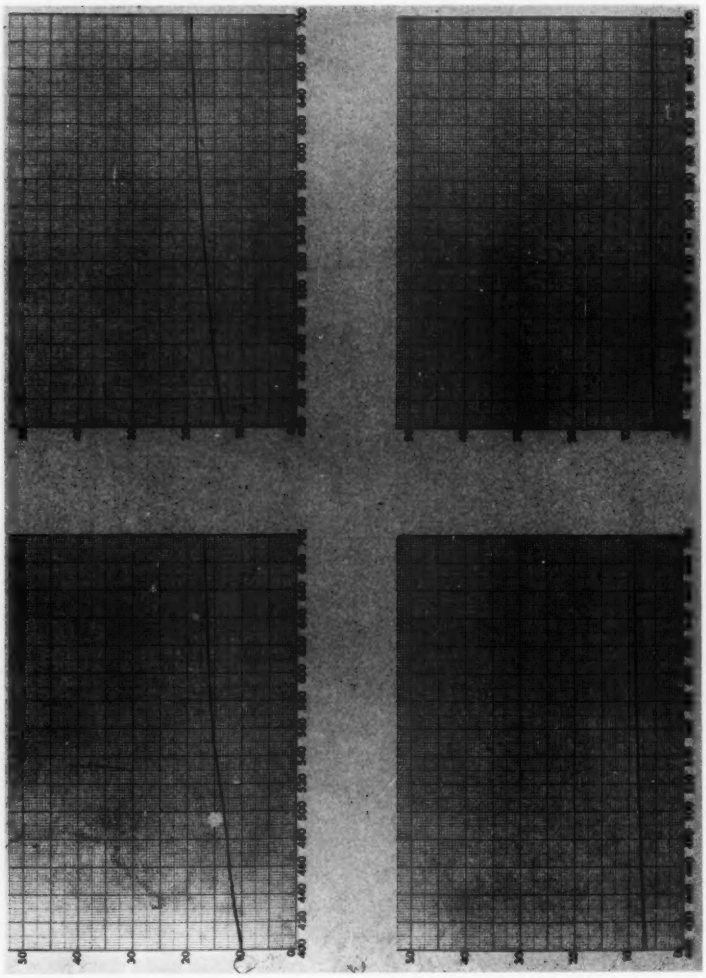

FIGURE 2
Spectral-reflectance curves for meteorites.
Parnallee Canyon Diablo
Farmington Mighei
Coördinates as in Figure 1.

checked against the national standard and found to be 0.98 of the incident light. The reflectance is accurate to 0.3 per cent and the scale of wavelengths to one Angstrom unit. The step-like variations at the blue end of

the curve for Waconda, the first specimen examined, resulted from a small instrumental time-lag which later disappeared. Other meteorites may be roughly placed on the reflectance scale by comparison with figure 3. The shades used are those which describe the surfaces examined and are comparable to the usual scale of shades employed in the description of mete-

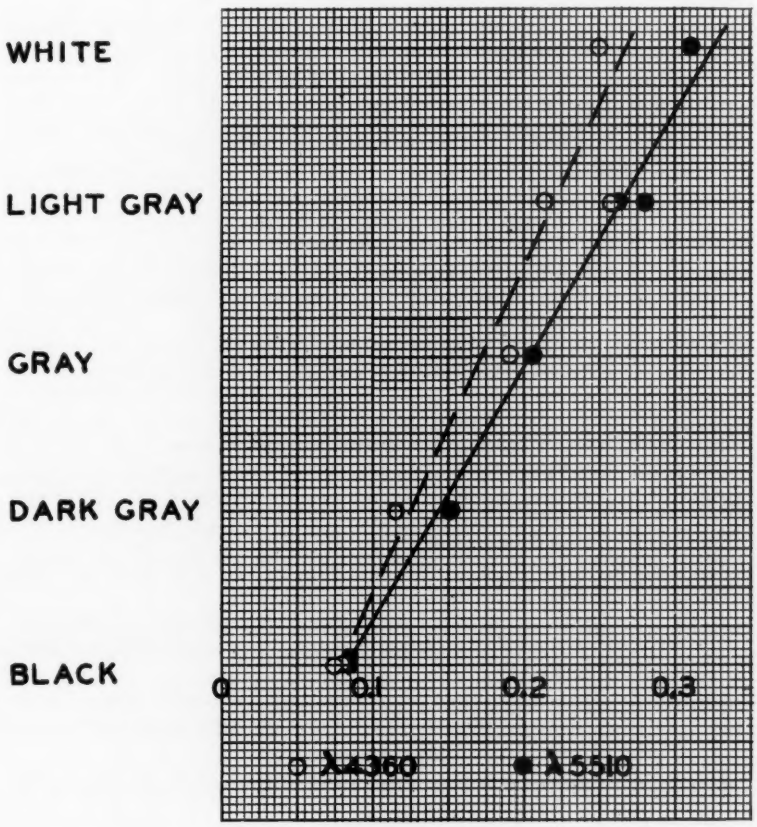

FIGURE 3
Reflectance values for meteorites of different shades.

orites. The flatness of the reflectance curves for the rust-free specimens shows that they are essentially "gray-bodies" over the spectral region in which sunlight is most intense. Thus the assumptions involved in the calculation of black- or gray-body temperatures for these masses in space are justified.

Color indices may be derived from the reflectance curves. The effective wave-lengths customarily used in the establishment of such indices are: photographic λ 4360, photovisual λ 5510 and red λ 6320. For these wave-lengths the reflectance values are transformed into color and red indices as given in table 2. Uncertainties in the intensities will not influence any indices except those of Farmington, which may be uncertain by 0.01 or 0.02 magnitudes. The three samples most colored are those most stained by rust and therefore they cannot be considered as representative. The other stone meteorites have color indices of approximately +0.10 magnitude. The color and reflectance values for the sample of Canyon Diablo suggest that on the basis of such data metallic and stony materials cannot be distinguished.

TABLE 1

DESCRIPTION OF SPECIMENS EXAMINED

STONE METEORITES	
Allegan	Michigan (Specimen 480a), fell July 10, 1899, 8 A. M. Oransite chrondrite, very friable. Surface rough, many chondri, no rust stains, considered as light gray.
Drake Creek	Nashville, Tennessee (Specimen 105b) fell May 9, 1827, 4 P. M. Veined white hypersthene-chondrite, some rust stains, considered as white.
Farmington	Kansas (Specimen 439c), fell June 25, 1890, 1 P. M. Black hypersthene-chondrite, few metallic inclusions, no rust stains, considered as black.
Mighei	Ukraine (Specimen 552), fell June 18, 1889, 8:30 A. M. Carbonaceous chondrite, sawn surface examined, some light chondri, no rust stains, considered as black.
Olmedilla de Alarcon	Spain (Specimen 580), fell February 26, 1929. Gray brecciated chondrite, no rust stains, considered as gray.
Parnallee	Madras, India (Specimen 233e), fell February 28, 1857, noon. Veined gray hypersthene-chondrite, many rust stains, considered as dark gray.
Waconda	Kansas (Specimen 344f), found 1873. Brecciated spherical hypersthene-chrondrite, a few rust stains, considered as light gray.
IRON METEORITE	
Canyon Diablo	Arizona (Specimen 440e), surface deeply etched after slight polish, little specular reflection.

To evaluate properly the data given by the reflectance curves we must partially describe the measuring machine. The photosensitive surface is essentially a sphere; the sample is placed over a small opening in it opposite to the incoming beam of parallel light. At various wave-lengths the intensity of reflected light is compared photo-electrically with that from a magnesium-oxide surface identically situated in a second chamber. The reflectance is the integration of all light reflected from a "flat" surface over

the hemisphere of phase angle less than ninety degrees. In the terminology of the Bond albedo used in astronomy[3] the reflectance is comparable to total albedo rather than to the "*p*-term," or geometrical albedo. Because of the rough, irregular surfaces of the meteorites examined it is likely that their "*q*-terms" are less than unity; 0.7 has been adopted provisionally, with recognition that the average asteroidal value is 0.55 (Russell, op. cit.). On the basis of this assumption geometrical albedoes, "*p*-terms," are given in table 2.

TABLE 2

REFLECTIVITY AND COLOR INDICES OF METEORITES

NAME	I_p	I_v	I_r	I_v/I_p	I_r/I_p	C. I.	R. I.	"p_{vis}"*
Allegan	0.257	0.280	0.284	1.09	1.11	+0^m.09	+0^m.11	0.40
Drake Creek	0.250	0.310	0.323	1.24	1.30	0.23	0.29	0.44
Farmington	0.075	0.085	0.087	1.13	1.16	0.13	0.16	0.12
Mighei	0.079	0.085	0.084	1.08	1.06	0.08	0.06	0.12
Olmedilla	0.190	0.205	0.205	1.08	1.08	0.08	0.08	0.29
Parnallee	0.115	0.151	0.163	1.31	1.42	0.29	0.38	0.22
Waconda	0.213	0.262	0.273	1.23	1.28	0.22	0.27	0.32
Canyon Diablo	0.143	0.172	0.186	1.19	1.28	0.19	0.27	0.25

* Computed on the assumption that $I_v = 0.7\ p_{vis}$.

Summary.—From the spectral-reflectivity curves for meteorites, seven stony and one metallic specimen, it is concluded that (1) meteorites are essentially gray-bodies with color indices less than +0.2 magnitude; (2) their geometrical albedoes probably range from 0.1 to 0.5; (3) metallic and stony materials are probably indistinguishable on the basis of color data.

[1] Schneiderhöhn and Ramdohr, *Lehrbuch der Erzmikroskopie*, **2**, 39 (1931).
[2] E. Krinov, *Russ. Astr. Journ.*, **14**, 356 (1937).
[3] Russell, *Ap. Jour.*, **43**, 101, 173 (1916).

ON THE DUPLEXITY THEORY OF VISUAL RESPONSE IN VERTEBRATES. II

BY W. J. CROZIER AND E. WOLF

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Communicated November 15, 1938

I.—The duplexity theory of visual performance in vertebrates is to the effect that retinal rods and their associated visual purple are connected with scotopic vision, indistinct and colorless, cones with photopic vision, distinctness and color. The predominance (partial or exclusive) of cones in the retinas of diurnal and of rods in nocturnal animals, and the presence of both in animals characteristically active both by day and night, is a very general rule.[1]

By use of the technique based upon response to visual flicker it has been found that the majority of vertebrates give a relationship between flash-frequency F and flash-intensity I critical for recognition of flicker[2] which includes 2 (in one case[3] 3) segments.

For the diurnal turtle *Pseudemys*, with purely cone retina, the $F - \log I_m$ curve occupies the general position of that for the "cone" segment of other vertebrates[2,4] (with allowance for temperature); the curve is in this case a simple probability integral over its whole extent. The testing of nocturnal animals with purely rod retinas provides material critical for the evaluation of certain general propositions of the duplexity theory in its traditional form. Such an animal is found in the gecko *Sphaerodactylus*.[5] The threshold response to flicker which is the basis for the measurements consists in a movement in the direction of the rotation of a striped cylinder;[6] the relation of flicker-recognition to other forms of intensity discrimination[7] indicates that the flicker-response contour is a delicate and precise test of general visual function.[8]

II.—The dissection of the duplex $F - \log I$ contour for most vertebrates can be made by use of the extrapolated probability integral which describes the upper or "cone" segment.[2,9] If the lower segment is to be assigned to the activities of retinal rods, in keeping with the elementary form of the duplexity doctrine, then we should expect that for a purely rod-retina animal one might obtain an $F - \log I$ contour rising rather slowly to a comparatively low maximum, and with an inflection point at a comparatively low flash-intensity. It might also be found that beyond a certain maximum level of F the curve then declines,[10] but there is evidence[11] that the falling off of the "rod" contribution to the duplex curve is more probably due to the inhibitory action of the "cone" effects.

The measurements made with *Sphaerodactylus* are collected in figure 1. The expansion of the iris[8] of the gecko eye below a flash-intensity of *ca.*

0.0002 millilamberts causes the critical intensity to be mechanically lowered. This is proved by the effect of instilled pilocarpine, which constricts the pupil at low intensities, and by the measurements with iris expanded by atropine at higher intensities (see figure 1).

The curve drawn through the observed points is a probability integral. Its maximum F (26.51 for I_m; 26.79 for F_m[12]) is low by comparison with

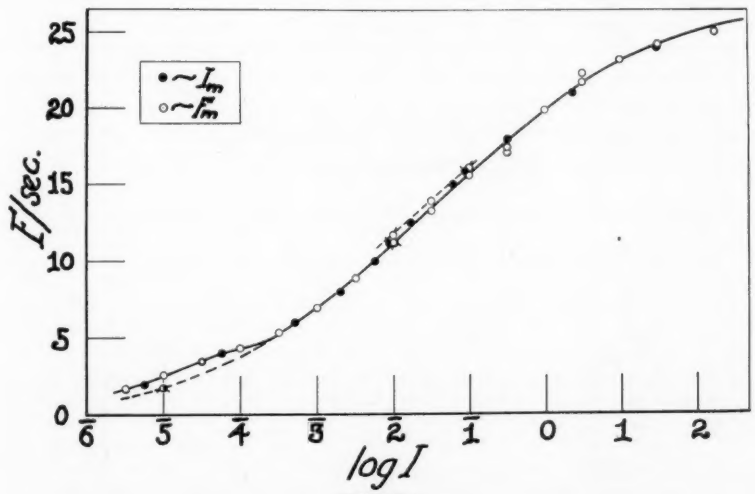

FIGURE 1

The flicker response contour for the gecko *Sphaerodactylus inaguae* Noble. Solid dots, log I_m at fixed flash-frequencies F; open circlets, F_m at fixed flash-intensities; circlets with tags on the *lower* side, determinations after instillation of pilocarpine solution (constricting the pupil); circlets with tags on the *upper* side, after instillation of atropine solution (opening the pupil). The full line with the lower dashed extension is a computed probability integral, which in the absence of the effect at the lower end (as proved by the influence of the constricted pupil) due to the opening of the pupil below *ca.* log $I = \bar{4}.3$, gives an excellent description of the data. Temperature, 26.7° ± 0.6°; flash-cycle with 50 *per cent* light time. At each point, 3 observations on each of 10 individuals; the data are not homogeneous, in the sense that the same 10 were not used for the different tests (hence the distinction[6] between I_m and F_m is not apparent).

that for all other animal types tested (47 to 63, under the same general conditions). Its inflection point, however, is at a high intensity close to those for the "cone" sections of the curves of other vertebrates.

III.—The lowness of $F_{max.}$ could be a consequence of the small size of the eye, which should also tend to increase the value of log I at the inflection. The comparison between the curves for a purely cone-[2,4] and purely rod-retina animal can be most clearly made by bringing them to

the same value of $F_{max.}$ (100 *per cent*). This will not affect log I at inflection ($= \tau'$), and permits comparing the spread constants $\sigma'_{\log I}$ since the form of the function is in each case that of a probability integral $F_{max.}$ is a specific property of the type of animal, but is not directly correlated with τ' or $\sigma'_{\log I}$.[13] This comparison is made in figure 2.

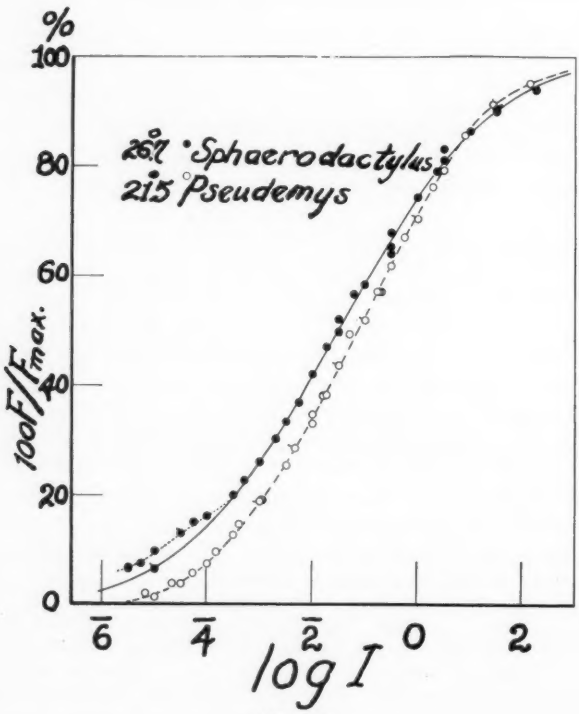

FIGURE 2

The flicker-response contours for *Sphaerodactylus* (gecko; *rod* retina) and *Pseudemys* (turtle; *cone* retina) compared by adjusting to a flash-frequency scale in which $F_{max.}$ for each = 100 *per cent*. The curves drawn are the computed probability integrals. The slight difference is further reduced by correcting for the difference in temperature (gecko, 26.7°; turtle, 21.5°). The standard deviation ($\sigma'_{\log I}$) is a little greater for *Sphaerodactylus*.

The gecko covers the same range of intensities in its $F - \log I$ curve as does the turtle. The temperature was different in the two cases; applying the temperature characterisitic found for the decrease of τ' with rise of temperature in *Pseudemys*,[14] the values of τ' for gecko and the turtle differ

only by -0.08 log units. The values of $\sigma'_{\log I}$ differ by less than do those for different genera of fishes.[13]

IV.—These observations clearly give no support to the conception that a rod retina necessarily functions best at low illuminations, or less ably at higher illuminations than a cone retina,[15] or that the excitabilities of rods are restricted to lower illuminations. They cast increased doubt upon the validity of specifically associating histological appearance and functional capacity. In the analysis of visual responses it is to be recognized that the occurrence of two component populations of effects in the constitution of the response contour signifies merely that these must be separately considered on the basis of the quantitative descriptions of their properties.[2] There need not necessarily be involved any direct correlation of these sets of properties with histological differentiations of the peripheral receptive field.

A fuller account of the experiments will shortly be published elsewhere.

[1] Scharrer, E., *Zeits. vergl. Physiol.*, **11**, 104 (1929); Verrier, M.-L., *Bull. Biol. Fr. Belg.*, **71**, 238 (1937).

[2] *Proc. Nat. Acad. Sci.*, **24**, 125 (1938).

[3] *Jour. Exptl. Zoöl.* (in press) [*Triturus*].

[4] *Jour. Gen. Physiol.* (in press) [*Pseudemys*].

[5] We are greatly indebted to Mr. R. A. McLean for a supply of these geckos collected at Matthew Town, Great Inagua, Bahama Islands; and to Dr. Thomas Barbour, Director of the Museum of Comparative Zoölogy, for his identification of them as *Sphaerodactylus inaguae* Noble.

[6] *Jour. Gen. Physiol.*, **19**, 495 (1935–1936); **20**, 211, 393, 411 (1936–1937).

[7] *Ibid.*, **19**, 503 (1935–1936); **20**, 393 (1936–1937); **21**, 313 (1937–1938).

[8] On the gecko retina, see: Rochon-Duvigneaud, A., *Ann. d'oculist.*, Nov., 35 (1917); Detwiler, S. R., *J. Comp. Neurol.*, **36**, 125 (1923).

[9] *Jour. Gen. Physiol.*, **20**, 411 (1936–1937); **21**, 17 (1937–1938); etc.

[10] *Ibid.*, **20**, 411 (1936–1937).

[11] *Ibid.* (in press) [Related fishes, II] (1938–1939).

[12] *Ibid.*, **20**, 211, 363 (1936–1937).

[13] *Proc. Nat. Acad. Sci.*, **23**, 516 (1937); **24**, 542–545 (1938); *Jour. Gen. Physiol.*, **21**, 17 (1937–1938); (in press).

[14] *Proc. Nat. Acad. Sci.*, **24**, 216 (1938).

[15] The relation of these findings to the hypothesis advanced by Walls that the nocturnal geckos are descended from ancestors with cone-retinas (Walls, G. L., *Amer. J. Opthalmol.*, **17**, 892 (1934)) is of course entirely a matter for speculation.

SPECIFIC CONSTANTS FOR VISUAL EXCITATION. III

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Communicated November 15, 1938

I.—Curves are obtainable in a precisely reproducible way which define the dependence of mean flash-intensity I_m critical for response to visual flicker upon the flash-frequency F.[1] For various animals these curves are of the same fundamental form, and are accurately described by a probability integral[2] ($F - \log I_m$). The properties of the three parameters of this formulation ($\sigma'_{\log I}$; abscissa of inflection, τ'; and maximum, $F_{max.}$), as revealed by their respective relations to temperature, retinal area and proportion of light-time in the flash cycle, are specifically consistent with this interpretation. The fundamental form is modified in most arthropods as a result of the convexity of the optic surface.[4] With most vertebrates it is complicated by the existence of two sets of elements of neural effects, having different mean log I thresholds,[5] so that the $F - \log I$ curve is then typically a complex of two overlapping sigmoid sections (which can be analytically separated[6]).

The specific differences between the $F - \log I$ curves for different kinds of animals gives opportunity to make a significant test of the invariant character of the parameters of the probability function. In forms with which adequate tests have been made thus far (turtle, sunfish, nymph of dragonfly) this specific invariance of $F_{max.}$ and shape constant $\sigma'_{\log I}$ under conditions producing change of τ', and the invariance of $\sigma'_{\log I}$ under conditions changing both τ' and $F_{max.}$ are striking evidence[3] for the propriety of the probability formulation. This is especially the case since in each of these instances the particular modification found is called for by the statistical conception of the basis for the expression of sensory effect.[7]

II.—The proof that simple constitutionally determined properties of the organism govern the expression of this sort of dynamical invariance can be obtained (but in a given test is not necessarily to be given in an obvious way) by means of genetic experiments.[8] For this purpose fresh water teleosts provide exceptional material, since they permit intergeneric cross breeding. Thus the $F - \log I_m$ curves for *Xiphophorus helleri* (*X.*, swordtail) and *Platypocoelius maculatus* (*P.*, platy) are quite different;[8,9] hybrid descendants of their cross breeding (H') show definite inheritance of certain characteristics (shape constants) of the $F - \log I_m$ curve from one or the other parent.[8, 9] These tests also demonstrated that the two populations of excitation-effects ("rod" and "cone") could be genetically separated; so likewise that the mechanisms determining τ' and $F_{max.}$ can be genetically influenced in independent ways.

III.—The importance of such facts in furthering the establishment of methods for the recognition of measures of biologically significant invariant properties led to a repetition of our earlier experiment.[8] First generation hybrids were obtained (H'') of two other species of the genera previously used, *X. montezuma* and *P. variatus*.

Adequate comparisons[10] showed that within each genus the flicker-response performance of several types each of genera *X.* and *P.* gave essentially the same $F - \log I_m$ curve—even in the case of albino *X. helleri*.[11] Figure 1 exhibits the curves for *X.*, *P.* and their F_1 hybrids (H''). Just as in the earlier (H') experiment[8] the upper segment of H'' is of precisely the same shape as that for *X.* The maximum for H'' is slightly higher, but $\sigma'_{\log I}$ is identical (Fig. 2). As with H', τ' is intermediate. For the lower

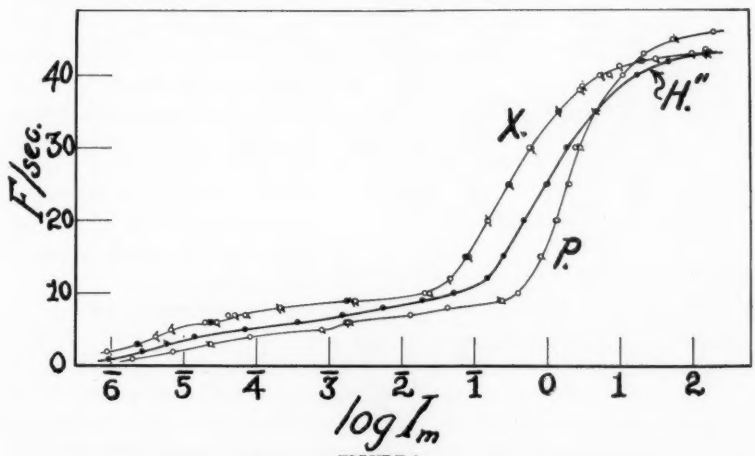

FIGURE 1

Flicker response contours for several types of *Xiphophorus* (*X.*), distinguished by different symbols, and including *X. helleri* and *X. montezuma;* for several varieties of *Platypoecelius* (*P.*), including *P. maculatus* and *P. variatus;* and for F_1 hybrids (*H."*) between *X. montezuma* and *P. variatus*. Temperature 21.5°, flicker-cycle with 50 per cent light time. For *X.* and *P.* the various series of plotted points are in different cases averages of 9 to 36 determinations each; for *H."* each plotted point is the mean of 9 (3 individuals).

("rod") segment, H'', $\sigma'_{\log I}$ corresponds to that for *P.*, not *X.* as with H' (cf.[9]), and $F_{max.}$ and τ' are intermediate (Figs. 1, 3).

IV.—The essential facts demonstrated by the analysis of the data in figure 1 agree with those found in our earlier experiment. The quantitative differences between $F_{max.}$, τ' in H' and H'' may be due to the fact that the parent species were different in the two cases, or to the fact that H' was a (uniform) back-cross product. These differences are at the moment

interesting only because they supply an internal check on the precision of the method of observation.

The results show why it is essential to have contours of measurements of performance over a wide range of the governing variable before classification of individuals is possible, or before genetic interpretations can be unambiguous.[12] They also illustrate a method of testing the relevance of

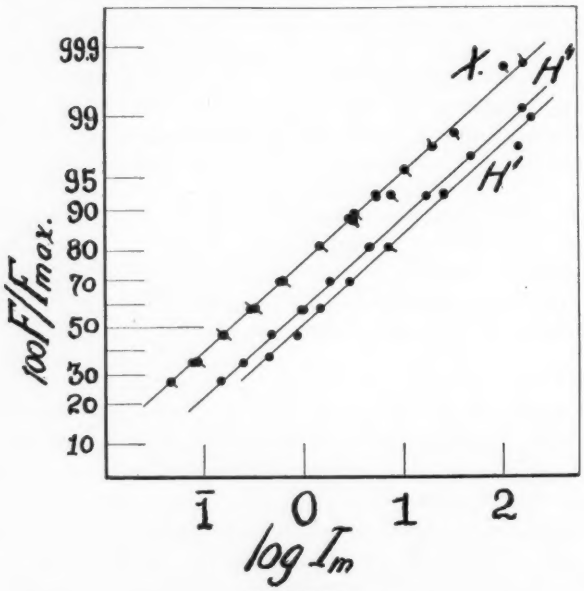

FIGURE 2

Data for *Xiphophorus* (*X.*) and for 2 types of its hybrids with *Platypoecelius* (*H'* and *H''*); only the upper or "cone" parts of the graphs in figure 1 are shown; *H'* is taken from a previous report.[10] The three lines are drawn parallel on a probability grid; $\sigma'_{\log I}$ is consequently the same—that for *P.* is of course much smaller. The maxima are respectively $F_{max.}$ equals for *X.* 43.05, *H'* 43.00, *H''* 43.34.

parameters implicit in analytical formulations of biological properties. In the nature of the present data[7] such formulations cannot be tested by curve fitting alone. The physical properties of the constants present must be suitable, and quantitatively appropriate.[3] Genetic tests indicate independent properties of these parameters, as called for by experimental results with different variables in tests with individual organisms.[3] They also show that determinate constitutional features of organization govern the exhibition of dynamical properties in biologically different animals. The properties

concerned in the invariant *form* of the flicker-response contour therefore must be properties arising from the fact that in different organic types physically different substrata produce dynamically similar results. These properties must therefore be essentially statistical.

What various kinds of animals have in common with respect to the invariant form of the flicker-contour is the possession of a large assemblage of (essentially variable) excitation elements. The parameters of their frequency distributions of effectiveness are fixed by the specific genetic composition, and must consequently depend upon the intrinsic properties of the physical substrata of these elements. This is entirely consistent with the directly determined control of the magnitudes of these parameters by temperature, area, and light-time cycle-fraction in different animals.[3] The same argument applies to the demonstration of the existence of two populations of visual sensory effects in most vertebrates, correlated with the presence of both retinal rods and cones.[9]

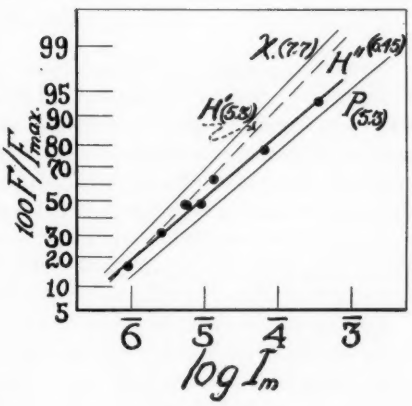

FIGURE 3

Data for $X.$, $P.$, H' and H'' in the lower or "rod" section of the flicker contour (cf. figure 1), are shown on a probability grid, with maxima as indicated giving best rectilinearity. The lines for $X.$, $P.$ and H' are taken from previous work.[10]

A more detailed consideration of these data is in course of publication.[10]

[1] *Proc. Nat. Acad. Sci.*, **23**, 71, 516 (1937); **24**, 125 (1938).

[2] *Ibid.*, **24**, 125 (1938); *J. Gen. Physiol.* (in press).

[3] *Jour. Gen. Physiol.*, **21**, 223 (1937–1938); **20**, 393, 411 (1936–1937); **21**, 313, 463 (1937–1938) (in press); *Proc. Nat. Acad. Sci.*, **24**, 216 (1938).

[4] *Jour. Gen Physiol.*, **21**, 223 (1937–1938); (1938–1939) (in press).

[5] *Ibid.*, **20**, 411, etc., (1936–1937).

[6] *Ibid.*, **20**, 411 (1936–1937); **21**, 313, etc. (1937–1938).

[7] *Ibid.*, **19**, 503 (1935–1936); *Proc. Nat. Acad. Sci.*, **22**, 412 (1936); **23**, 23, 130 (1937).

[8] *Ibid.*, **23**, 516 (1937).

[9] *Jour. Gen. Physiol.*, **21**, 17 (1937–1938).

[10] *Ibid.* (in press).

[11] *Proc. Nat. Acad. Sci.*, **24**, 221 (1938).

[12] *Jour. Gen. Physiol.*, **13**, 57, 81, etc. (1929–1930).

THE EFFECTS OF CHEMICALS ON THE LETHAL MUTATION RATE IN DROSOPHILA MELANOGASTER

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Communicated November 14, 1938

Numerous attempts have been made to induce mutations artificially in Drosophila by means of chemical agents. The early procedures involved feeding of larvae chemical substances mixed with the food.[2-6] Various poisons, acids and bases, and dyes all gave negative results. Later, Stadler[7] and Medvedev[8] showed that the salts of heavy metals, in conjunction with the application of x-rays, produced a significant increase in the mutation rate over that resulting from x-raying alone. This was in barley and Drosophila, respectively. In these cases, however, the substances probably only increased the effectiveness of absorption.

The Russian workers were the first to report indications of gene mutations, resulting from treatment of fertilized Droposhila eggs and larvae with various chemical substances. Lobašov and Smirnov[9] reported an increase in the lethal mutation rate in Drosophila, the larvae of which were treated with NH_4OH. Lobašov[10] also presented data indicating an increased lethal mutation rate due to treatment of larvae and pupae with acids, although Goldat and Beliaieva[11] could find no significant increase from egg treatment with HCl.

More recently a group of workers of the Bubnov Pedagogical Institute in Moscow reported increased mutation rates resulting from treatment of fertilized Drosophila eggs, chorion intact, with I_2 in KI, $CuSO_4$ and $KMnO_4$[12-14]. Preliminary data by Samjatina and Popowa[15] and Kondakowa[16] also indicated that iodine treatment produced mutations, although the lethal mutation rate (second chromosome) reported by the latter was extremely high for a relatively small number of chromosomes tested, $7.8 \pm 2.7\%$ for 121 chromosomes tested and 9.1 ± 1.7 for 285 chromosomes tested. More recently Lobašov[17] found an increase in the mutation rate, lethal and visible, through injection of various strengths of NH_4OH and HCl into Drosophila pupae.

In plants Stubbe[18] has reported the induction of mutations in Antirrhinum resulting from several specific chemical substances.

In some of the experiments reported above the strains of Drosophila used were not given, the incidences of lethal mutations were of doubtful significance, and there appeared as a result of the chemical treatment in some cases an abnormally high percentage of both lethal and visible mutations. Thus, in view of the fact that both negative and positive results are reported for the same substances, it seemed worth while, first, to use a

TABLE 1

THE EFFECTS OF VARIOUS CHEMICALS ON THE LETHAL MUTATION RATE IN THE *X*-CHROMOSOME OF THE OREGON-R RACE OF DROSOPHILA MELANOGASTER

EXP. NO.	CHEMICAL	TYPE	TREATMENT CONC.	TIME	NO. OF MALES	NO. OF *X*-CHROME. TESTED	NO. OF LETHALS	% LETHALS	LOCATION OF LETHALS	MUTATION RATE
A	$CuSO_4$	Injection	0.1%		6	471	3, 2[s]	1.06		
2A	$CuSO_4$	Immersion of eggs	Conc.	10 min.	5	480	3, 3[s]	1.25	I, II[s], II[s]	
		Total			11	951	6, 5[s]	1.16		1 in 86.4
B	$KMnO_4$	Injection	0.025%		4	405	..	..		
2B	$KMnO_4$	Immersion of eggs	Conc.	15 min.	4	447	1, 2[s]	0.66	I, II[s], II[s]	
		Total			8	852	1, 2[s]	0.35		1 in 284
C	I_2 in KI	Injection	0.038% I 0.075% KI		6	386	2[s]	0.52		
2C	I_2 in KI	Immersion of eggs	50%	5 min.	3	460	1[s]	0.22	I[s]	
		Total			9	846	3[s]	0.35		1 in 282
D	Colchicine	Injection	0.0013%		4	421	..	..		
2D	Colchicine	Immersion of eggs	Conc.	4 min.	4	305	..	..		
3D	Colchicine	Immersion of eggs	0.5%	15 min.	3	259	..	..		
		Total			11	985	..	..		0 in 985
E	Ringers (Control)	Injection			7	507	..	..		0 in 507*

s = semi-lethal mutations. Those producing a sex ratio between 1:0.30 and 1:0. Only females not carrying ClB are considered, and only culture vials containing at least 50 flies were used. Lethals are located as I—between *sc* and *v*, and II—between *v* and *f*.

* Demerec[21] found 2 lethals in 3049 *X*-chromosomes tested in this stock.

new technique for administering the chemicals, and second, to remove the chorion from fertilized eggs before treatment. The positive results published by the Bubnov authors were obtained through treatment of chorion-intact fertilized eggs.

In one set of experiments the chemical substances were injected into larvae by use of a micro-injection needle, following the method of Beadle and Ephrussi.[19] In this manner from 0.5 to 1 cubic mm. of the substance may be injected into each larva. The gonads were thus bathed in the solution. For each chemical substance used the sub-lethal dosage was determined by injection of a series of concentrations into appropriate test larvae. In the second series of experiments the chorion was removed from fertilized eggs by gently stroking with a blunt glass needle. These were immersed in sub-lethal dosages of the chemicals. These dosages were determined by running a series of different concentrations over different time intervals. The various sub-lethal dosages employed in this study are listed in table 1.

I_2 in KI, $CuSO_4$, $KMnO_4$ and colchicine have been used in testing effects of the lethal mutation rate. The first three substances were used because of their reported influence of

TABLE 2

COMPARATIVE RESULTS OF THE LETHAL MUTATION RATE OBTAINED BY TREATMENT WITH VARIOUS CHEMICALS

CHEMICAL USED	TYPE	TREATMENT CONC.	TIME	NO. OF SEP. EXPS.	NO. OF X-CHROME. TESTED	NUMBER LETHALS	% LETHALS	DIFF. $\frac{\%}{\sigma_D\ \%}$	AUTHOR
$CuSO_4$	Eggs immersed	Conc. aqueous	40 min.	1	579	6	1.03	0.82 ± 0.29†	Magrzhikovskaja, 1936
$CuSO_4$	Eggs immersed	Conc. in alcohol of various strengths	5–40 min.	5	2826	27, 16*	1.52	1.31 ± 0.25	Magrzhikovskaja, 1936
$CuSO_4$	See table 1			2	951	6, 5*	1.16	1.16 ± 0.48	This report
$KMnO_4$	Eggs immersed	Conc. aqueous	5–80 min.	4	1101	7	0.63	0.39 ± 0.46	Naumenko, 1936
$KMnO_4$	Eggs immersed	Conc. alcohol	5 min.	1	582	(24)* 9	(4.12) 1.55	1.39 ± 0.53	Naumenko, 1936
$KMnO_4$	See table 1			2	852	1, 2*	0.35	0.35 ± 0.25	This report
I_2 in KI	Eggs immersed	5% in 10% KI	15–20 min.	1	703	7	0.99	0.55 ± 0.49	Ssacharow, 1936
I_2 in KI	See table 1			2	846	3*	0.35	0.35 ± 0.25	This report

† All errors are standard errors.

* See text.

gene mutation and colchicine because of its apparent effect of mitotic processes.[20]

The inbred Oregon-R stock has been used as test animals. Only the *X*-chromosome was studied, and lethals were detected by the usual ClB method:

(1) ClB/x-ple* female by +(treated) male

(2) F_1 of ClB/+ female by x-ple male

(3) Females from cultures indicating lethal or semi-lethal mutations were mated by x-ple males.

Flies for (1) were raised in $^1/_2$ pint milk bottles, and for (2) and (3) in 25 × 95 mm. vials. The standard food of corn meal, molasses and agar, with brewers' yeast added, was used throughout.

Eggs were collected over a two-hour period at 25°C. The eggs used for the second series of experiments were approximately from 2 to 4 hours old when treated. For the injection series larvae were grown in culture dishes rich in yeast, and larvae just prior to pupation were used.

From table 1 it can be seen that there has resulted an increase in the lethal mutations rate of the *X*-chromosome from treatment with $CuSO_4$ in both series of experiments, injection of a 0.1% solution and treatment of naked, fertilized eggs with a concentrated aqueous solution for 10 minutes. The percentage of lethals obtained in each group was approximately the same. The ratio of the percentage difference to the standard error of the percentage difference for both groups combined is 2.5. The ratios reported by Magrzhikovskaja[12] for chorion-intact, fertilized eggs, treated with concentrated aqueous and alcoholic solution for longer time periods, were somewhat higher. See table 2.

The lethal mutation rate obtained in the manganese series is relatively higher than in the Ringer's injected controls, but not significantly so. A total of 3 lethals was obtained in 852 tested chromosomes, none in the injection series. In the concentrated aqueous $KMnO_4$ series reported by Naumenko[13] there is likewise a greater percentage of lethals reported in the experimental group, but again not a significant increase in the rate. However, in his series treated with concentrated $KMnO_4$ in alcohol the lethal mutation rate is significantly greater than the control group. This holds true if a correction is made of his reported mutation rate 4.12 per cent. Of 24 lethals obtained, 18 appeared in 3 groups originating from 3 parents. The corrected rate should read 1.55 per cent (table 2).

Although Kondakowa and Popowa and Samjatina reported data suggesting that iodine treatment may produce mutations, the treatment resorted to in this study, injection of 0.038 per cent I_2 in 0.075 per cent KI, and immersion of naked eggs in a 50 per cent solution of I_2 in KI for 5 minutes did not produce a significant increase in the lethal mutation rate of the *X*-chromosome. Ssacharow,[14] also, failed to induce mutations in

the X-chromosome through treatment of fertilized eggs with I_2 in KI, but did find a significant increase in the mutation rate of the third chromosome.

It is of particular interest that of the lethals obtained in the copper series using 11 treated males there appeared no genetically identical mutations. In the case of egg treatment it might be expected that an effect of the chemical should be produced on germ cells of a very early embryonic stage, and in the case of injections into mature larvae that some grouping of lethals might occur due to the effect of the chemical on some spermatagonial progenitor, providing, of course, that all affected cells survive. Magrzhikovskaja, likewise, reported no grouping of the 43 lethals he obtained in the copper series.

No lethal mutations were obtained in a total of 985 tested X-chromosomes coming from males receiving injections of 0.0013 per cent colchicine or through egg treatments with concentrated and 0.5 per cent aqueous solutions.

Summary.—$CuSO_4$ is found to influence the lethal mutation rate of the X-chromosome in *Drosophila melanogaster*, either by injection into mature larvae or treatment of naked, fertilized eggs with sub-lethal dosages.

Both $KMnO_4$ and I_2 in KI bring about a slight increase in the lethal mutation rate, but these are not statistically significant increases.

A comparative study is made with the results obtained by other workers using these same chemical substances.

Colchicine is found to be ineffective in altering the lethal mutation rate although it does reportedly influence the mitotic processes.

[1] Parker Fellow from Harvard University to Stanford University in 1937–38.

[2] Morgan, T. H., *Amer. Nat.*, **48** (1914).

[3] Mann, M. C., *Jour. Exptl. Zoöl.*, **54** (1923).

[4] Muller, H. J., these PROCEEDINGS, **14** (1928).

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[9] Lobašov, M., and F. Smirnov, *Ibid.*, Ser. **A2**, 307 (1934).

[10] Lobašov, M., *Trav. Soc. Natur. Leningrad*, **63**, 271 (1934).

[11] Goldat, S. J., and V. N. Beliaieva, *Biol. Žurn*, **4**, 384 (1935).

[12] Magrzhikovskaja, K. V., *Bull. Biol. Med. Exp.*, **I** (1936).

[13] Naumenko, B., *Ibid.*, **I** (1936).

[14] Ssacharow, W. W., *Genetica*, **18** (1936).

[15] Samjatina, N. D., and O. T. Popowa, *B. Th.*, **3** (1936).

[16] Kondakowa, A. A., *Biol. Žurn*, **4**, 721 (1935).

[17] Lobašov, M., *Genetica*, **19**, 2001 (1937).

[18] Stubbe, H., *Angewandte Chemie*, **50**, 241 (1937).

[19] Method of Beadle, G. W., C. W. Clancy, and Boris Ephrussi, *Proc. Roy. Soc. Lon.*, Series B, No. 826, **122** (1937).

[20] See Nebel, B. R., and M. L. Ruttle, *Jour. Hered.*, **29**, 1, 3 (1938) for early literature.

[21] Demerec, M., *Genetics*, **22**, 469 (1937).

* The stock designated x-ple carries in the X-chromosome the mutant genes *sc*, *ec*, *cv*, *ct*^{6}, *v*, *g*^{2}, *f* in that order.

AN INTERNAL SECRETION AFFECTING VIABILITY IN CRUSTACEA

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Aside from the work of Koller (1930) in which he found that an eyestalk hormone appeared to function in the control of calcium deposition in the exoskeleton of crustaceans, nothing other than the chromatophoric activities of these hormones has been suggested. During the course of earlier work upon the effects of removal of eyestalks from *Palaemonetes vulgaris* upon the state of pigment dispersion in the chromatophores, I frequently observed that the eyestalkless animals gradually lost their glassy transparency over the course of a few days and could be readily distinguished from the normal animal in this respect. Equally conspicuous was the lack of loss of transparency when the blinding operation consisted only of the removal of the ommatidial portion of the stalk leaving the major portion of the stalk tissue intact. The difference in effects of these two types of operations was correspondingly registered in a completely different type of chromatophore behavior. At the time, a possible extra-chromatophoric activity of the eyestalk hormones was not followed up.

With the discovery of glands comparable with the eyestalk glands of decapod crustaceans in lower crustaceans which are not equipped with functional pigment cells (Stahl 1938) and in the head regions of such other arthropods as the Insecta (Hanström 1935), it has suggested itself more and more strongly that the chromatophoric hormones of crustaceans, or substances closely related to them in place of formation, have other and fundamental functions in the general body metabolism. It is the purpose of this brief report to describe some initial experiments that have been performed upon crayfishes and which demonstrate quite conclusively the importance of an eyestalk hormone in the normal continuation of the life of the individual.

During the course of such experiments five species of crayfishes have been used with substantially similar results with each. The particular experiments reported here, however, are the most extensive ones and were conducted with the species *Cambarus blandingi acutus*. These animals ranged in carapace length from 12 to 30 mm.

The first experiment involved treating animals in four different ways. One lot of crayfishes had both eyestalks removed, another had the eyestalks removed but with the stalk tissue implanted into the ventral abdominal region, a third lot suffered removal of a single eye, while the fourth lot was entirely untreated. The animals were placed in individual finger bowls

in which the water was about $1^1/_2$ cm. deep. The bowls were loosely covered with glass plates.

A second experiment was performed to determine the exact extent to which the length of postoperative life could be increased by the transplantation procedure. For this experiment two lots of crayfishes were subjected to eyestalk removal; the first lot was given eyestalk tissue injected into the ventral abdominal region in Ringer's solution; the second lot had a similar quantity of clear Ringer's solution injected in a similar manner.

In a third experiment crayfishes after removal of the eyestalks were placed in a refrigerator at a temperature of about 7 degrees Centigrade.

In all the experiments the operative procedure was quite simple. The eyestalks were removed at their base with a sharp, pointed scalpel and the wound promptly seared over with an electric cautery needle. When the animal was to receive an abdominal implant of eyestalk tissue, the two stalks were immediately placed in a drop of Ringer's solution and the chitinous exoskeleton removed from them with fine forceps. Care was taken to remove all the living tissue with the exception of bits of the hypodermis. The larger pieces of tissue were then teased to pieces and the suspension injected by means of a glass pipette into the ventral abdomen through an aperture made in the fifth or sixth segment. The pipette tip was pushed anteriorly so that the introduced tissue came to occupy a position in the first, second and third segments and was clearly visible through the transparent cuticle. The total volume of the injected mass seldom exceeded 0.05 cc. When an eyestalkless animal was to serve as a control for the effect of the implant, an injection of Ringer's solution was made in the same way as has just been described. With the use of such a control the results demonstrated a more decisive difference between eyestalkless animals with and without the tissue implant than in the first experiment where the difference in the severity of the operations tended to draw together the averages of their lengths of life. The effect of the abdominal injection itself seems to decrease the average postoperative life by about one day.

Table 1 indicates the results of each of the above experiments, giving the frequency distribution of the deaths of the animals over a period of days. In all of the experiments those animals which died within the first twenty-four hours after the operation (about 20 per cent) were not included in the data. The great majority of these deaths were attributable to operative shock or excessive loss of blood resulting from unsuccessful cautery of the stubs. Further justification for the omission of these was found in the distribution of deaths over a period of days. Neglecting the deaths of the first day the times of the deaths displayed fairly normal distribution. Among other advantages such omission permitted more accurate determination of the significance among the means.

Several facts are immediately evident as a result of these experiments. The eyestalks are essential to the normal continuation of life of these crayfishes, those animals having them removed dying in less than half the time

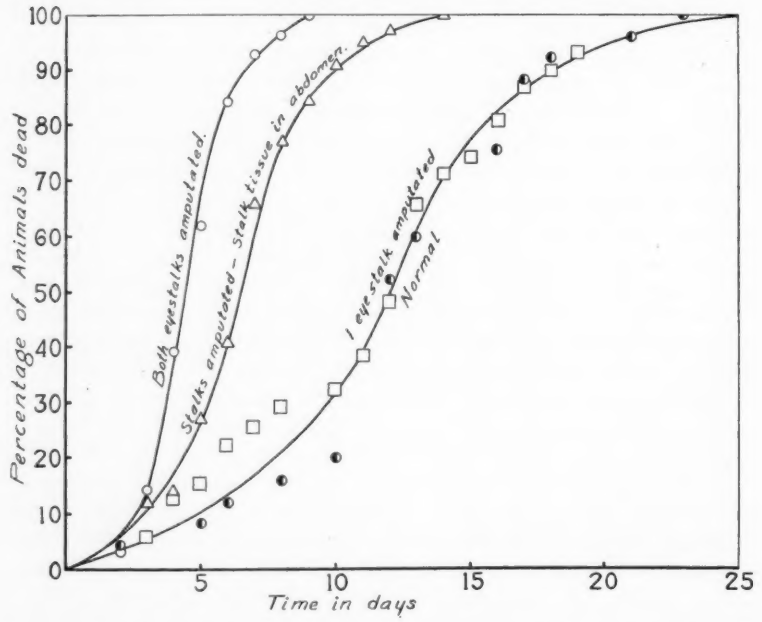
TABLE 1

			EXP. I		EXP. II		EXP. III
DAYS	NORMAL	1 EYE OFF	2 EYES OFF	2 EYES OFF INJ.	2 EYES OFF RINGER'S	2 EYES OFF EYE TISS.	2 EYES OFF 7°C.
2	1	1	3	..	..	..	..
3	..	1	4	4	4	1	..
4	..	2	10	..	7	1	..
5	1	1	11	5	4	1	2
6	1	2	14	4	1	2	2
7	..	1	6	7	..	4	..
8	1	1	2	3	..	2	..
9	..	..	3	3	..	..	1
10	1	1	..	2	..	1	1
11	..	2	..	1	..	..	..
12	8	3	..	1	..	1	..
13	2	5	..	..	..	..	..
14	..	2	..	..	..	1	..
15	3	1	..	..	..	..	..
16	1	3	..	..	..	..	..
17	3	2	..	..	..	..	1
18	1	1	..	..	..	..	..
19	..	1	..	..	..	..	..
20	..	1	..	..	..	..	..
21	1	..	..	..	..	..	..
22	..	..	..	..	..	..	..
23	..	..	..	..	..	..	1
24	1	..	..	..	..	..	1
25	..	..	..	..	..	..	..
26	..	..	..	..	..	..	1
27	..	..	..	..	..	..	..
28	..	1	..	..	..	..	..
29	..	..	..	..	..	..	..
30	..	..	..	..	..	..	..
31	..	..	..	..	..	..	1
32	..	..	..	..	..	..	..
33	..	..	..	..	..	..	..
41	..	..	..	..	..	..	1
50	..	..	..	..	..	..	1
Av. Survival	13.1	11.9	5.3	6.3	4.1	7.5	20.0
S. E.	∓0.94	∓0.98	∓0.23	∓0.42	∓0.22	∓0.81	

taken by normal ones. The definiteness of the necessity of the eyestalks for continued life is furthermore shown in the small standard error of the average length of postoperative life. On the other hand, it appears that a single remaining stalk is sufficient to maintain life over practically the

normal life span, there being no evidence in these results of a significant difference between the means of the normal animals and those from which a single eyestalk has been removed.

The implantation experiments lead strongly to the conclusion that the influence of the eyestalks upon viability is through a chemical control rather than because of any positional or structural relationship with the rest of the body. Even shredded bits of tissue in the abdominal region are capable of very significantly prolonging the life of eyestalkless animals. Thus far there has been no histological examination of the region of the implant to determine to what extent incorporation has gone on, or what particular



tissues of the stalks, if any, have persisted. When this is done it is possible that very interesting conclusions can be deduced as to the actual tissues involved in the activity. There is, however, some physiological evidence that there has been some incorporation of the elements involved in the production of the substance in question. This is seen in figure 1 which is a summary of experiments one and two combined. If the effects of the injection were one of a chemical substance introduced into the blood only at the time of the actual injection, then one would perhaps expect that there would be a more marked difference between the injected and the uninjected during the early postoperative period and then after the substance had

disappeared from the blood there would be equal rates of dying. That is not the case. The results indicate that some element is most effective in lowering the death rate after four or five days have elapsed since the injection. This finds ready explanation in the assumption of incorporation and subsequent activity of the implanted tissue.

The results thus far described pertain to experiments conducted at room temperature of about 25 degrees centigrade. Temperature has a very marked effect upon the length of time the animal is able to survive without eyestalks, as would be expected. Eyestalkless crayfishes kept at 7 degrees lived on the average of about 20 postoperative days as compared with the 5.3 days for ones kept at room temperature. The $Q10$ would thus be about 2.1.

It has perhaps already been noted that the normal animals lived in the conditions of the experiment only 12 days on the average. This shortness of life is undoubtedly the result of the absence of food coupled with a relatively high temperature and is assumed to be the time for starvation. None of the animals were fed during the course of the experiments. Being kept in separate containers, they were also unable to feed upon one another as they frequently do in the stock supply.

An attempt was made to determine whether there was any difference between sexes as to resistance to eyestalk removal. There appeared to be no difference whatsoever. Another attempt at correlation, that between size of the individual and resistance, appeared to indicate that through quite a range of sizes (carapace lengths from 12 to 30 mm.) there was none, but that animals somewhat larger than these dimensions were somewhat less affected by removal of the stalks. Too few of the larger animals have yet been used to establish this definitely.

It is much too early even to guess what relationship this vital substance bears to the chromatophore hormones of the eyestalk either with regard to chemical similarity or identity, or point of origin. However, there is definitely a chemical substance produced somewhere in the crayfish eyestalk essential to continued life of the animal.

Hanström, B., "Uber eine Substanz im Insektenkopf, die zusammenballend auf das Pigment der Garneelenkromatophoren wirkt," *Kungl. Fys. Sallsk. Lund Forhandl.*, **6**, No. 8 (1935).

Hanström, B., "Vermischte Beobachtungen über die Chromatophoraktivierenden Substanzen der Augenstiele der Crustaceen und des Kopfes der Insekten," *Kungl. Fys. Sallsk. Handl.*, **47**, No. 8 (1937).

Koller, G., "Weitere Untersuchungen über Farbwechsel und Farbwechselhormone bei Crangon vulgaris," *Zeit. vergl. Physiol.*, **12** 632–667 (1930).

Stahl, F., "Über das Vorkommen von inkretorischen organen und Farbwechselhormonen im Kopf einiger Crustaceen," *Kungl. Fys. Sallsk. Handl.*, **49**, No. 12 (1938).

THE CONSISTENCY OF THE AXIOM OF CHOICE AND OF THE GENERALIZED CONTINUUM-HYPOTHESIS

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THEOREM. *Let* T *be the system of axioms for set-theory obtained from v. Neumann's system* S*[1] *by leaving out the axiom of choice* (Ax. III 3*); *then, if* T *is consistent, it remains so, if the following propositions* 1–4 *are adjoined simultaneously as new axioms:*

1. The axiom of choice (i.e., v. Neumann's Ax. III 3*)
2. The generalized Continuum-Hypothesis (i.e., the statement that $2^{\aleph_\alpha} = \aleph_{\alpha+1}$ holds for any ordinal α)
3. The existence of linear non-measurable sets such that both they and their complements are one-to-one projections of two-dimensional complements of analytic sets (and which therefore are B_2-sets in Lusin's terminology[2])
4. The existence of linear complements of analytic sets, which are of the power of the continuum and contain no perfect subset.

A corresponding theorem holds, if T denotes the system of Princ. Math.[3] or Fraenkel's system of axioms for set theory,[4] leaving out in both cases the axiom of choice but including the axiom of infinity.

The proof of the above theorems is constructive in the sense that, if a contradiction were obtained in the enlarged system, a contradiction in T could actually be exhibited.

The method of proof consists in constructing on the basis of the axioms of T[5] a model for which the propositions 1–4 are true. This model, roughly speaking, consists of all "mathematically constructible" sets, where the term "constructible" is to be understood in the semiintuitionistic sense which excludes impredicative procedures. This means "constructible" sets are defined to be those sets which can be obtained by Russell's ramified hierarchy of types, if extended to include transfinite orders. The extension to transfinite orders has the consequence that the model satisfies the impredicative axioms of set theory, because an axiom of reducibility can be proved for sufficiently high orders. Furthermore the proposition "Every set is constructible" (which I abbreviate by "A") can be proved to be consistent with the axioms of T, because A turns out to be true for the model consisting of the constructible sets. From A the propositions 1–4 can be deduced. In particular, proposition 2 follows from the fact that all constructible sets of integers are obtained already for orders $<\omega_1$, all constructible sets of sets of integers for orders $<\omega_2$ and so on.

The proposition A added as a new axiom seems to give a natural completion of the axioms of set theory, in so far as it determines the vague notion of an arbitrary infinite set in a definite way. In this connection it is important that the consistency-proof for A does not break down if stronger axioms of infinity (e.g., the existence of inaccessible numbers) are adjoined to T. Hence the consistency of A seems to be absolute in some sense, although it is not possible in the present state of affairs to give a precise meaning to this phrase.

[1] Cf. *J. reine angew. Math.*, **160**, p. 227.

[2] Cf. N. Lusin, *Leçons sur les ensembles analytiques*, Paris, 1930, p. 270.

[3] Cf. A. Tarski, *Mh. Math. Phys.*, **40**, p. 97.

[4] Cf. A. Fraenkel, *Math. Zeit.*, **22**, p. 250.

[5] This means that the model is constructed by essentially transfinite methods and hence gives only a relative proof of consistency, requiring the consistency of T as a hypothesis.

THE CHARACTERIZATION OF PSEUDO-$S_{n,r}$ SETS

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I. If to each pair of elements (*points*) p, q of an abstract set there is attached a non-negative real number (*distance*) pq, independent of the order of the elements, while $pq = 0$ if and only if $p = q$, the resulting space is called *semimetric*. A fundamental problem in the distance geometry of the n-dimensional spherical surface $S_{n,r}$ of radius r(the n-dimensional surface of a sphere of radius r in euclidean space of $n + 1$ dimensions, with "*shorter arc*" distance) consists in characterizing those semimetric spaces S which have the following properties: (1) S contains more than $n + 3$ points, (2) if $p, q \epsilon S$, then $pq \neq d = \pi r$, (3) if $p_1, p_2, \ldots, p_{n+2}$ are elements of S, then there exists a function f mapping these $n + 2$ points upon $S_{n,r}$ with preservation of distances (i.e., *congruently*), (4) S cannot be mapped congruently upon a subset of $S_{n,r}$. Reserving the details of the investigation for publication elsewhere, we summarize in this note the complete solution of this problem. Semimetric spaces S with properties (3), (4) are called pseudo-$S_{n,r}$ sets.[1]

The properties of the $S_{n,r}$, by virtue of which the characterization theorems of pseudo-$S_{n,r}$ sets are obtained, are all consequences of the following *metric* ones: (1) the mutual distances of each set of $n + 2$ points of $S_{n,r}$ satisfy a relation of the form $\left| \varphi(p_i p_j / r) \right| = 0$, $(i, j = 1, 2, \ldots, n + 2)$, where $\varphi(pq/r)$ is a real, single-valued, monotonically de-

creasing function defined over the *distance set* of $S_{n,r}$, with $\varphi(0) = 1$ and $\varphi(d/r) = -1$, ($\varphi(pq/r) \equiv \cos pq/r$), (2) for each integer k, and each set of $k + 1$ points of $S_{n,r}$, the above determinant is non-negative, while there exists a set of $n + 1$ points for which the determinant is positive, (3) each set of $n + 1$ points with a non-vanishing determinant forms a *complete metric basis*, (4) if $p_1, p_2, \ldots, p_n, p$ are $n + 1$ points of $S_{n,r}$, with non-vanishing determinant, there exists at least one point p', distinct from p, such that $pp_i = p'p_i$, $(i = 1, 2, \ldots, n)$.

II. *Characterization Theorems.*—The two following characterization theorems are obtained.

FIRST CHARACTERIZATION THEOREM. *If* P *is a pseudo-*$S_{n,r}$ *set containing more than* n + 3 *points and without diametral points (i.e., no pair of points of* P *has a distance equal to* d), *then every pair of distinct points of* P *has the distance* $r \cdot \mathrm{Cos}^{-1}(\pm 1/(n+1))$. *Not every distance equals* $r \cdot \mathrm{Cos}^{-1}(1/(n+1))$.

SECOND CHARACTERIZATION THEOREM. *Let* P *be a pseudo-*$S_{n,r}$ *set containing more than* n + 3 *points and without diametral points. Then for every positive integer* k, *the determinant* $|\cos p_i p_j / r|$, (i, j = 1, 2, . . . , k + 1), *formed for* k + 1 *points of* P *has* (*upon multiplication of appropriate rows and the same numbered columns by* −1) ALL *elements outside the principal diagonal equal to* $-1/(n+1)$.

Thus, a pseudo-$S_{n,r}$ set containing more than $n + 3$ points, and without diametral points, is *essentially* equilateral, with distance $r \cdot \mathrm{Cos}^{-1}(-1/(n+1))$. It is, of course, obvious that a set of arbitrary power exceeding $n + 2$ and with every distance equal to $r \cdot \mathrm{Cos}^{-1}(-1/(n+1))$ is a pseudo-$S_{n,r}$ set without diametral points, but that *every* pseudo-$S_{n,r}$ set *containing more than* n + 3 *points*, and without diametral points, has essentially this simple structure is surprising. Many interesting conclusions follow from this result.

[1] For $n = 1$ the term pseudo-*d*-cyclic is used. The characterization of pseudo-*d*-cyclic sets has been given (Blumenthal, L. M., *Amer. Jour. Math.*, **54,** 387–396, 729–738 (1932); **56,** 225–232 (1934) but the treatment for higher dimensions demands quite different methods. The case $n = 2$ is solved in *Distance Geometries*, University of Missouri Studies, 1938. The general case involves several departures from the procedure adopted there.

ON SEMI-GROUPS OF SELFADJOINT TRANSFORMATIONS IN HILBERT SPACE

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In a paper dealing with the representations of Lie groups by linear transformations in the n-dimensional and Hilbert space,[1] I have obtained the following particular result:

Let H_t $(-\infty < t < \infty)$ *be a family of bounded selfadjoint transformations in Hilbert space* $\mathfrak{H}$, *forming a group, i.e.,* $H_0 = E$ (*the identical transformation*), $H_tH_s = H_{t+s}$. *Suppose that* (*for every element* f *in* $\mathfrak{H}$) *the numerical function* $(H_tf, f) = |H_{t/2}f|^2$ *of* t *is Lebesgue-measurable in a suitable neighborhood of* $t = 0$. *Let us denote by* m *resp.* M *the lower resp. the upper bound of* H_1 (*we have* $m > 0$). *Then there is a resolution of the identity,* E_λ, *continuous from the left, such that* $E_\lambda = 0$ *for* $\lambda \leqq m$, $E_\lambda = E$ *for* $\lambda > M$ *and that*

$$H_t = \int_0^\infty \lambda^t \, d\, E_\lambda. \tag{1}$$

The same conclusion holds if we suppose boundedness, instead of measurability, of the function (H_tf, f) *in a neighborhood of* $t = 0$.

Recently, E. Hille has considered the case of families H_t $(0 < t < \infty)$ of positive definite selfadjoint transformations forming semi-groups, i.e., such that $H_sH_t = H_{s+t}$ for $s > 0$, $t > 0$.[2] Making the assumption that the upper bounds of the $H_t - E$ are less than, or equal to, 1, and using a theorem on the Laplace-Stieltjes representation of absolutely monotone functions, he gets a spectral representation for H_t essentially of the form of (1).

In this short note I shall show how a result on *semi-groups*, containing that of Hille's, can be obtained by the same method as used in my paper. This result reads as follows:

Let H_t $(0 < t < \infty)$ *be a family of founded selfadjoint transformations in* $\mathfrak{H}$, *forming a semi-group, i.e.,* $H_sH_t = H_{s+t}$ *for* $s > 0$, $t > 0$. *Suppose that* (*for every element* $f \epsilon \mathfrak{H}$) *the numerical function* $(H_tf, f) = |H_{t/2}f|^2$ *of* t *satisfies in a certain interval* $a \leqq t \leqq b$ (a *and* b *may depend upon* f) *at least one of the following conditions: 1) it is bounded, 2) it is measurable. Denote by* m *and* M *the lower and upper bounds of* H_1 *respectively* (*as* $H_1 = (H_{1/2})^2$, *we have* $m \geqq 0$). *Then there is a resolution of the identity,* E_λ, *continuous from the left, such that* $E_\lambda = 0$ *for* $\lambda \leqq m$, $E_\lambda = E$ *for* $\lambda > M$ *and*

$$H_t = \int_0^\infty \lambda^t dE_\lambda.$$

Demonstration.—Using Schwarz's inequality, we see that the function $h_f(t) = \log (H_t f, f)$ (f arbitrary, but fixed) is a convex function of t on $0 < t < \infty$:

$$2h_f((t+s)/2) = \log(H_{(t+s)/2} f, f) = \log (H_{t/2} f, H_{s/2} f) \leqq \log (|H_{t/2} f|^2 \cdot |H_{s/2} f|^2) = h_f(t) + h_f(s).$$

We know further that $h_f(t)$ is bounded from above, or that it is measurable, in a certain interval $a \leqq t \leqq b$. But a convex function enjoying in a subinterval of its domain one of these properties is necessarily continuous in the whole interior of its domain.[3] $h_f(t)$ and therefore $(H_t f, f)$ are thus continuous for $0 < t < \infty$. As f was arbitrary, H_t must vary itself continuously with the parameter t.

Let now E_λ be the resolution of the identity corresponding to H_1; we have then $E_\lambda = 0$ for $\lambda \leqq m$, $E_\lambda = E$ for $\lambda > M$ (m and M denoting the bounds of H_1, $m \geqq 0$). Putting

$$G_s = \int_0^\infty \lambda^s dE_\lambda \qquad (s > 0),$$

we get a new semi-group of bounded selfadjoint transformations, $G_1 = H_1$. H_t being permutable with H_1, is also permutable with G_s. Putting $(G_{1/2} - H_{1/2})f = g$, we have

$$|G_{1/4} g|^2 + |H_{1/4} g|^2 = (G_{1/2} g, g) + (H_{1/2} g, g) = ((G_{1/2} + H_{1/2}) (G_{1/2} - H_{1/2}) f, g) = ((G_{1/2}^2 - H_{1/2}^2) f, g) = ((G_1 - H_1) f, g) = 0.$$

Thus $G_{1/4} g = H_{1/4} g = 0$ and therefore also $G_{1/2} g = H_{1/2} g = 0$. It follows that $|(G_{1/2} - H_{1/2}) f|^2 = ((G_{1/2} - H_{1/2})^2 f, f) = ((G_{1/2} - H_{1/2}) g, f) = 0$, $G_{1/2} = H_{1/2}$. Repeating these arguments, we can verify the identity of H_t and G_t for $t = 1/2^n$ ($n = 0, 1, 2, \ldots$). The semi-group property yields then this identity for $t = m/2^n$ ($m, n = 1, 2, 3, \ldots$). Finally, as a result of the continuity, we get $H_t = G_t$ for all values of t.

[1] Béla v. Sz. Nagy, "Über messbare Darstellungen Liescher Gruppen," *Mathematische Annalen*, **112**, 286–296 (1936).

[2] Einar Hille, "On Semi-Groups of Transformations in Hilbert Space," *Proc. Nat. Acad. Sci.*, **24**, 159–161 (1938).

[3] Cf. J. L. W. L. Jensen, "Sur les fonctions convexes etc.," *Acta Mathematica*, **30**, 175–193 (1906); W. Sierpiński, "Sur les fonctions convexes mesurables," *Fundamenta Mathematicae*, **1**, 125–129 (1920).

GROUPS OF ORDER p^m CONTAINING $m - \alpha$ INDEPENDENT GENERATORS

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If a group of order p^m, p being a prime number, contains a set of $m - \alpha$ independent generators then every set of its independent generators involves exactly $m - \alpha$ operators, and the cross-cut of its subgroups of index p is invariant, of the order p^α and the corresponding quotient group is abelian and of type $1^{m-\alpha}$. We shall here confine our attention to the case when this cross-cut is cyclic. When the group G is abelian there is one and only one such group in view of the obvious theorem that every abelian group of order p^m is completely determined by m and the cross-cut of its subgroups of index p. This group in the present case is of type $\alpha + 1$, $1^{m-\alpha-1}$ and hence its properties are well known. In what follows it will therefore be assumed that G is non-abelian unless the contrary is stated. It is obvious that G cannot involve any operator whose order exceeds $p^{\alpha+1}$ since the pth power of each of its operators appears in each of its subgroups of index p. When p is odd and $m > 2$ it is known that there is at least one group of order p^m which has the property that it involves no operator whose order exceeds p and that each of its subgroups of index p involves a given subgroup of order p. In this group $\alpha = 1$ but the group contains no operator of order $p^{\alpha+1}$.

It is easy to prove that in every other case G involves operators of order $p^{\alpha+1}$. In the special case when $p = 2$ it is well known that not all the operators of G besides the identity can be of order 2 since it is assumed that G is non-abelian. If G contains no operator of order $2^{\alpha+1}$ it contains a subgroup of order $2^{\alpha+2}$ which involves a commutator subgroup of order 2^α and hence contains an operator which transforms a commutator of order 2^α into its inverse. The given subgroup of order $2^{\alpha+2}$ therefore involves an abelian subgroup of order $2^{\alpha+1}$ which gives rise to a commutator of order 2^α under this subgroup of order $2^{\alpha+2}$. As this is impossible it has been proved that G involves operators of order $2^{\alpha+1}$ when $p = 2$. When $p > 2$ it results directly that it would involve a subgroup of index p which would not include the given cyclic subgroup of order p^α, $\alpha > 1$, if it did not contain an operator of order $p^{\alpha+1}$. Hence there results the following theorem: *If a group of order* p^{m} *has the property that the cross-cut of all its subgroups of index* p *is the cyclic group of oder* p^α *then it has* $\mathrm{m} - \alpha$ *independent generators and involves operators of order* $\mathrm{p}^{\alpha+1}$, *whenever* $\alpha > 1$, *but no operator of a higher order.* In the special case when $p = 2$, it is not necessary to assume that $\alpha > 1$.

Since the case when $p = 2$ involves various considerations which do not present themselves when p is an odd prime number it will be assumed in what follows that p is odd unless the contrary is stated. The commutator subgroup of G is of order p since the pth transform of every operator of G with respect to the powers of a given operator is the operator itself. All the operators of order p contained in G, together with the identity, constitute an invariant subgroup of G and all of the operators of G whose orders divide a given power of p less than $p^{\alpha+1}$ constitute such a subgroup which is of index p under the group generated by the operators of G whose orders divide the next larger power of p. Since the cross-cut of all the subgroups of index p contained in G is generated by every operator of order $p^{\alpha+1}$ contained in G and these operators of order $p^{\alpha+1}$ generate G it results that this cross-cut appears in the central of G. It results from the dihedral group of order 2^m, $m > 3$, that this theorem does not in general apply to the case when $p = 2$. Some of the other theorems of this paragraph do also not apply to this special case which is excluded for the present.

Since G is assumed to be non-abelian it must include operators of order p which are non-invariant, for if all its operators of order p would be in its central then all its operators whose orders divide p^α would be invariant, but this is impossible since the central quotient group of every group is non-cyclic. It results from the quaternion group that this theorem is also not generally true when $p = 2$. Let s_1 be a non-invariant operator of order p contained in G and considered the subgroup of index p under G composed of all of its operators which are commutative with s_1. If this subgroup involves an operator of order $p^{\alpha+1}$ then G involves an operator s_2 of order p which is non-commutative with s_1 and the two operators s_1, s_2 generate the non-abelian group of order p^3 which involves no operator of p^2. If the subgroup of index p composed of all the operators of G which are commutative with s_1 does not involve any operator of order $p^{\alpha+1}$ then it is either the abelian group of type α, $1^{m-\alpha-1}$ or it involves a non-invariant operator of order p.

In the former case all the operators of order $p^{\alpha+1}$ contained in G are non-invariant and G is completely determined, being constructed by adjoining to the given abelian group an operator of order $p^{\alpha+1}$ which transforms one of its independent generators of order p into itself multiplied by a commutator of order p. If this subgroup of index p contains no operator of order $p^{\alpha+1}$ and is non-abelian it contains two non-invariant operators s_3 and s_4 which together generate the non-abelian group of order p^3 which involves no operator of order p^2 since the given subgroup of index p contains invariant operators of order p^α. Hence two distinct cases present themselves. In one of these G contains invariant operators of order $p^{\alpha+1}$ while in the other it contains no invariant operator whose order exceeds p^α. It will be convenient to consider these two cases separately, beginning with

the former. The method of procedure will be to construct an infinite system of groups coming under each of these two cases and then to prove that such a system is composed of all the possible groups which come thereunder.

Starting with the abelian group of type $\alpha + 1, 1^\beta$ we extend this group by an operator t_1 of order p which is commutative with each of the operators in a reduced set of its independent generators except with one t_2 of those of order p. If $\beta > 1$ we may repeat this process any arbitrary number of times not exceeding β times. It is obvious that each of the groups thus obtained satisfies the conditions imposed on G and hence the total number of such distinct groups is $(m - \alpha - 1)/2$ when $m - \alpha$ is odd and exceeds unity and $(m - \alpha - 2)/2$ when $m - \alpha$ is even and exceeds 2. The smallest group in this category is the non-abelian group of order p^4 which involves a cyclic central of order p^2 but no operator of order p^3. It is easy to see that every G whose central involves an operator of order $p^{\alpha+1}$ is a group contained in this system. In particular, when $m - \alpha$ *is an odd positive number greater than unity there are* $(m - \alpha - 1)/2$ *groups of order* p^m, p *being an odd prime number, which separately have the properties that the cross-cut of all their subgroups of index* p *is the cyclic group of order* p^α *and that their centrals involve operators of order* $p^{\alpha+1}$. *When* $m - \alpha$ *is an even positive number there are* $(m - \alpha - 2)/2$ *such groups*. In both cases each of these groups has $m - \alpha$ independent generators.

If we start with the abelian group of type $\alpha, 1^\beta$ we can similarly extend this group when $\beta > 0$ by an operator s_1 of order $p^{\alpha+1}$ which has its pth power in this abelian group and is commutative with each of the operators in a reduced set of its independent generators except with one s_2 of those of order p which it transforms into itself multiplied by one of its operators of order p which are generated by one of its operators of order $p^{\alpha+1}$. When $\beta > 1$ we can extend the group thus obtained by an operator N_3 of order p which is commutative with every operator in the given set of independent generators of the said abelian group of type $\alpha, 1^\beta$ except with one s_4 of order p which differs from s_2 and which it transforms into itself multiplied by the same commutator of order p as s_2 was transformed. The subgroup generated by s_3 and s_4 is the non-abelian group of order p^3 which involves no operator of order p^2, and the subgroup generated by s_1 and s_2 is the non-abelian group of order $p^{\alpha+2}$ which involves operators of order $p^{\alpha+1}$. All the operators of one of these two subgroups are commutative with every operator of the other. When $\beta > 2$ this process may be repeated an arbitrary number of times not exceeding β times.

The smallest group which belongs to this category is the non-abelian group of order p^3 which involves operators of order p^2. It is obvious that the direct product of an abelian group of type 1^k and any one of the groups noted above satisfies the conditions imposed on G and that when $m - \alpha$

is an even number the number of the distinct groups which come under the second of these categories is $(m - \alpha)/2$ and when $m - \alpha$ is an odd number the number of the distinct groups which come thereunder is $(m - \alpha - 1)/2$. Hence the groups of order p^m which have the property that the cross-cut of all their subgroups of index p is the cyclic group of order p^α present comparatively little difficulty when p is an odd prime number. In the special case when $\alpha = m - 1$ it is well known that there is only *one* such non-abelian group but when this cross-cut is of order p and $m > 3$ there are two categories of such groups composed respectively of those groups which have invariant operators of order p^2 and those which involve no such operators. In general there are two and only two groups for a given value of m and a given central whenever the order of the central quotient group is a given even power of p. When it is a given odd power of p there is no such group.

When $p = 2$ and the cross-cut of all the subgroups of index 2 contained in G is a cyclic group of order 2^α two cases present themselves. In one of these G is generated by its operators of order $2^{\alpha+1}$ and hence the given cyclic subgroup of order 2^α is in the central of G and the commutator subgroup of G is of order 2. This case is similar to the case considered above when p is odd and hence requires no special treatment here. It will therefore be assumed in what follows that the operators of order 2^α in the given cyclic subgroup of order 2^α are non-invariant under G and hence $\alpha > 1$. Since each of the cyclic subgroups of order $2^{\alpha+1}$ contained in G is invariant under G the given operators of order 2^α are transformed into their inverses under G and hence G contains a subgroup of index 2 in which the operators of the given cyclic subgroup of order 2^α are invariant. This subgroup is generated by the operators of order $2^{\alpha+1}$ contained in G and hence is either abelian or has a commutative subgroup of order 2.

In the former case it is of type $\alpha + 1, m - \alpha - 2$ and the remaining operators of G may transform all the operators of this subgroup into their inverses and thus give rise to two groups. In one of these all of the remaining operators are of order 2 while in the other all of these operators are of order 4. There is one additional group in which the remaining operators of G transform every operator of this abelian group into its $2^\alpha - 1$ power. In this case one-half of these additional operators are of order 2 while the rest are of order 4. In each of these cases the corresponding commutator subgroup is the cyclic group of order 2^α and each of the operators of order 2 in the given abelian subgroup of index 2 under G corresponds to itself. Another isomorphism of this group with respect to the cyclic group of order 2 can be established whenever the order of the given abelian group exceeds $2^{\alpha+1}$ by letting some of the operators of order 4 correspond to themselves. Hence there is another G which involves the given abelian group as a subgroup of index 2. In this group half of the remaining operators

are also of order 2 while the rest are of order 4. There are therefore four such G's which involve this abelian subgroup of index 2.

It remains to consider the case when the given subgroup of index 2 has a commutator subgroup of order 2 but when the operators of order 2^α in the cross-cut of all the subgroups of index 2 under G are not invariant under G. Since each of the cyclic subgroups of order $2^{\alpha+1}$ contained in G is again invariant under G the given operators of order 2^α are transformed into their inverse by half of the operators of G. The remaining operators of G are therefore of orders 2 and 4, since when all the operators of a group which do not appear in a subgroup of index 2 are of order 2 this subgroup is necessarily abelian. It is therefore necessary to consider subgroups of the given subgroup of index 2 which give rise to a cyclic quotient group of order 2^α with respect to this subgroup. These subgroups can be selected in various ways depending upon whether the given subgroup of index 2 involves invariant operators of order $2^{\alpha+1}$ or does not have this property. The number of distinct cases to be considered is somewhat large but the separate cases present little difficulty.

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